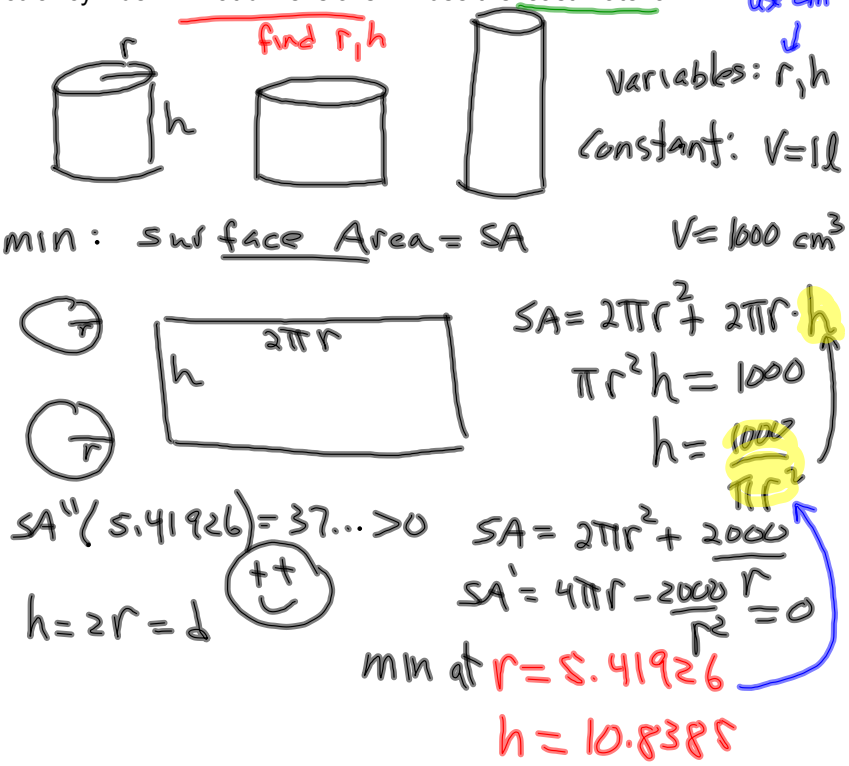
4.4b Modeling and Optimization

Designing a can

You have been asked to design a one-liter oil can shaped like a right-circular cylinder. What dimensions will use the least material?



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A cylindrical can holds a fixed volume. Find the ratio of height to radius for the can with least surface area.

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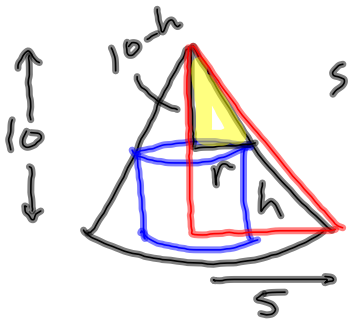
A cylinder is inscribed in a cone of height 10 and radius 5. Find the volume of the largest ~~cone~~ cylinder.

max volume



similar Δ's

$$2 = \frac{10}{5} = \frac{10-h}{r}$$

$$r = \frac{10-h}{2}$$

$$V = \pi r^2 h$$

$$V = \pi\left(\frac{10-h}{2}\right)^2 h$$

$$V' = \frac{(h-10)(3h-10)\pi}{4} = 0$$

$h = \frac{10}{3}$ max at ⇗

max is $V\left(\frac{10}{3}\right) = 116.355$

$V''\left(\frac{10}{3}\right) = -5\pi < 0$ ∩

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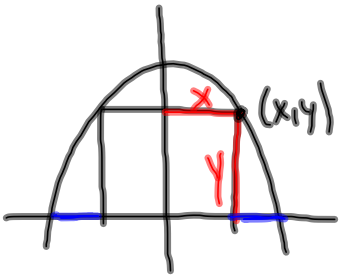
6.

$y = 12 - x^2$



$A = 2xy = 2x(12-x^2)$

$A = 24x - 2x^3$

$A' = 24 - 6x^2$ $x = \pm 2$

width = 4

height = 12 − 4 = 8

$A'' = -12x$ max at $x = 2$

$A''(2) = -24 < 0$ ∩

max area $A(2)$

$A = 4 \cdot 8 = 32$

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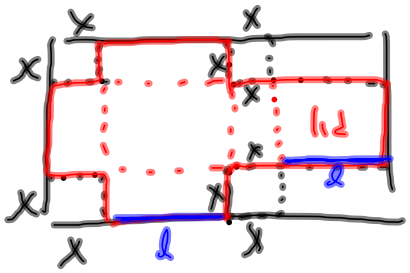
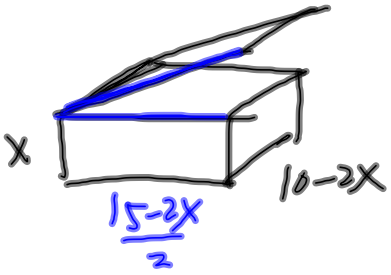
18.



10



15

$V = x\left(\frac{15-2x}{2}\right)(10-2x)$

$x + l + x + l = 15$

$2l = 15 - 2x$

$l = \frac{15-2x}{2}$

max at $x = 1.96187$

$V''(1.96187) = -26.45 < 0$

max vol is $V(1.96187) = 66.0191$

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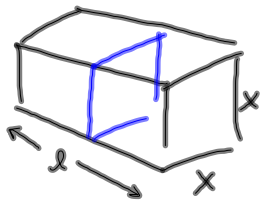
30.



girth $= g = 4x$

$l + g \le 108$

$l + 4x = 108$

max volume $= V = l \cdot x \cdot x$

$l = 108 - 4x$

$V = (108 - 4x)x^2$

$V = 108x^2 - 4x^3$

$V' = 216x - 12x^2 = 0$

$x(216 - 12x) = 0$

$x = 0 \quad 216 - 12x = 0$

$18 = \frac{216}{12} = 12x$

$V'' = 216 - 24x \Big|_{x=18} = -216 < 0$

dimensions

$18 \times 18 \times 36$

max at $x = 18$

max is $V = 108 \cdot 18^2 - 4 \cdot 18^3$

$V = 11{,}664$

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